DESIGNER'S GUIDE

op-amp feedback circuits Part 1

Feedback plots offer insight into operational amplifiers

Many engineers will benefit from a discussion of feedback effects and the construction of feedback plots for operational amplifiers. Part 1 of this series limits its discussion to single-stage amplifiers. Part 2 will conclude with an examination of multistage and composite amplifiers.

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Feedback plots simplify the analysis of an op amp's closed-loop ac performance by showing bandwidth and stability conditions as a function of the op amp's gain and phase response. These plots also provide insight into noise performance and the special feedback requirements of circuits such as integrating converters and photodiode amplifiers.

Engineers routinely use Bode plots (**Ref 1**) to determine the bandwidth and frequency stability of voltage-gain op-amp circuits. A Bode plot provides a visual representation of an op amp's transfer response and its potential stability. Moreover, such plots define the circuit's pole and zero locations at the intercepts of the response-curve extensions.

The Bode plot of **Fig 1**, for example, shows the interaction of the magnitude response of the open-loop gain (|A|) and the reciprocal of the feedback factor (1/β). The fraction of the output that feeds back to the input is β. The voltage-divider action of **Fig 1**'s feedback network determines the value of β; for moderate resistance values, $\beta = R_1/(R_1+R_2)$. For this noninverting example, the feedback equation, $A_{CL} = A/(1+A\beta)$, defines the closed-loop voltage gain. Aβ is the loop gain, and where it is high:

$$A_{CL} \approx 1/\beta = (R_1+R_2)/R_1.$$

Aβ represents the amplifier gain available to maintain the ideal closed-loop response. At the point where the loop gain no longer matches the feedback demand, the closed-loop curve deviates from the ideal. The Bode plot graphically defines this limit by plotting the 1/β curve with the gain-magnitude response curve of the op amp. Because the 1/β line represents the feedback demand, closed-loop requirements will be satisfied as long as this line is below the amplifier-gain curve. Where this condition is no longer true, the actual response drops, following the amplifier's open-loop response downward. The rate of descent for the roll-off is −20 dB/decade (for most op amps) and is characteristic of a single-pole response. In **Fig 1**, the heavier line on the gain-magnitude plot depicts the resulting closed-loop curve.

Back to basics

For a basic voltage-gain amplifier, the location of the f_p pole determines the closed-loop bandwidth. In this case, a single-pole roll-off determines the point at

Bode plots add a visual representation of the op amp's transfer response and provide information on gain, phase, and noise characteristics.

which the gain magnitude goes below 3 dB (equivalent to 0.707 of its low-frequency level). To find this point relative to the Bode plots, rewrite the closed-loop gain as

$$A_{CL} = (1/\beta)/(1/A\beta + 1).$$

The bandwidth-defining gain error is a result of the $1/A\beta$ term in the denominator. Because β is constant for the circuit in **Fig 1,** the amplifier gain (A) determines the frequency dependence of the loop gain. For a typical op amp, the gain-bandwidth product is constant after the first break frequency occurs and when

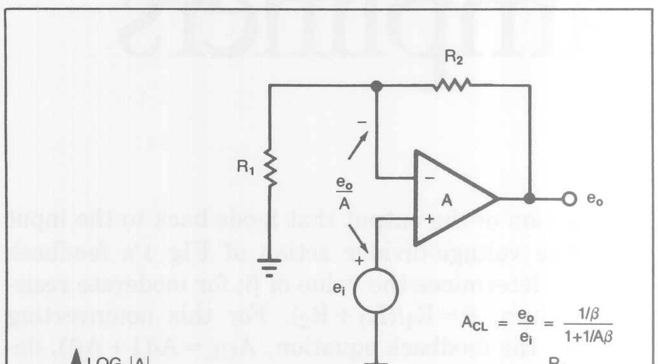

***Fig 1—This feedback analysis provides a summary of loop conditions in the 1/β curve** and defines the underlying poles, zeros, and phase shift.*

$A \approx jf_c/f = j|A|$ and where f_c is the amplifier's unity-gain crossover frequency. For this common condition,

$$A_{CL} = (1/\beta)/(1 + 1/(j|A|\beta)).$$

The bandwidth is defined in terms of the absolute value (magnitude) of A_{CL}:

$$|A_{CL}| = (1/\beta)/\sqrt{(1 + 1/(|A|^2\beta^2))},$$

which, at the −3-dB point, becomes

$$|A_{CL}| = 0.707(1/\beta) = (1/\beta)/\sqrt{2}$$

Comparing the last two expressions, you can see that the −3-dB bandwidth occurs when $|A| = 1/\beta$. This equality is true when the gain supply drops to the exact level of the feedback demand. When you plot these two functions on the same graph, they reach equality at the intersection of the two curves. This intercept pinpoints the closed-loop pole location and defines the circuit bandwidth for the voltage-gain amplifier.

When it comes to stability

This critical intercept point also exhibits other characteristics that can help you define conditions for frequency stability. By relating the phase shift to the slopes of the gain-magnitude and $1/\beta$ curves, you can determine the loop phase shift at this intercept. Again, the importance of the intercept is apparent from the closed-loop-gain expression, $A_{CL} = A/(1 + A\beta)$. If $A\beta$ becomes −1, the closed-loop gain will be infinite and will support an output signal even in the absence of an input signal, which is a condition for oscillation. The magnitude of $A\beta$ is unity only at the intercept point because it is at this point that $A = 1/\beta$; a negative polarity for $A\beta$ only requires 180° phase shift.

Virtually every practical analog circuit is a minimum-phase system. For such systems, which have only left-half-plane poles and zeros, you can directly read the phase shift from the gain-magnitude response **(Ref 2).** Although many op amps do have a right-plane zero caused by Miller phase compensation, the effects of this zero are suppressed below the unity-gain crossover. For the case of a minimum-phase system, a pole creates a −20-dB/decade response roll-off and a −90° phase shift; a zero produces the same effects but with opposite polarities. Additional poles and zeros simply add to the response slope and phase shift in increments of the same magnitude.

Relying on the feedback phase shift's correlation with the response slope, you can determine its value at the critical intercept from the gain-magnitude and $1/\beta$ curves. For the example of **Fig 1,** the gain-magnitude curve has a slope of -20 dB/decade and the $1/\beta$ curve has a zero slope for a net 90° feedback phase shift at the intercept. This situation leaves a phase margin of 90° out of the 180° that is needed to cause oscillation. Because the intercept is well removed from the open-loop-response break frequencies, the analysis of this example is easier to understand. The intercept occurs after the amplifier's first pole develops the full 90° phase shift, but well before the second pole has any effect.

Maximum error is 5.7°

In cases where the intercept is less than two decades from a response break, the Bode approximation of the phase shift shows a linear slope that has a maximum error of 5.7° **(Ref 1).** For **Fig 1,** the phase-shift approximation starts at 0° one decade before the break frequency f_o. From there, it increases linearly on the log scale to 45° at the break frequency and then to 90° one decade above it.

Using this approximation, you can combine the stability criteria for loop-gain magnitude and feedback phase shift to obtain the rate-of-closure indicator. Rather than computing phase shifts from slopes, you can use this indicator to deal with the slopes directly. Rate-of-closure is simply the difference in slopes of the gain-magnitude curve and the $1/\beta$ curve when they intercept. This difference reflects the combined phase shift around the feedback loop. For **Fig 1,** the rate-of-closure is 20 dB/decade, which corresponds to a stable 90° phase shift.

In other cases, the slope of the $1/\beta$ curve is not zero, giving a 40-dB/decade rate-of-closure that indicates an oscillatory 180° of phase shift. Rate-of-closure alone is an exact stability indicator where the intercept is at least two decades away from all other break frequencies. In still other cases, the Bode phase approximation modifies the rate-of-closure result.

Feedback network needs its independence

To use feedback relationships to perform circuit analysis, you should consider the feedback network separately. This separation parallels the nature of the op amp's open-loop gain, which is a characteristic of the amplifier in the absence of the feedback network. You only need to retain the loading effects between the amplifier and the feedback network to determine their individual responses **(Ref 2).** Then, by putting the two responses on the same plot, you can see how they will work together.

Fig 2 shows a generalized feedback condition defined by Z_1 and Z_2. The equations of **Fig 2a** directly determine the circuit response for high loop gain and moderate impedances. Nonetheless, the input impedance of the amplifier alters the simplified results of these equations by shunting the feedback network. The inclusion of this loading effect on the feedback network completes the $1/\beta$ analysis in the circuit of **Fig 2b.** Here, the op-amp input resistance (R_i), differential input capacitance (C_{id}), and common-mode input capacitance (C_{icm}) all shunt impedance Z_1. Except for conditions where the feedback impedances have low values, you need to include these amplifier characteristics in your analysis.

Where there is impedance in series with the amplifier's noninverting input, you must add this too—along with the shunting effect of the input's C_{icm} capacitance. You can then find the feedback factor from the divider action, e_j/e_o. For the $1/\beta$ curve, this result is inverted and, in the logarithmic format of computer simulations, becomes simply VDB(o) − VDB(j). By adding this curve to the plot of the amplifier's gain-magnitude response, you can display the characteristics of the critical intercept for subsequent feedback interpretation.

The feedback network's voltage-divider action determines the feedback factor, which interacts with the open-loop gain to generate the overall circuit response.

Gain departures deserve consideration

The $1/\beta$ curve also communicates performance information across the entire response range of the op amp. For example, it displays loop gain, which provides an

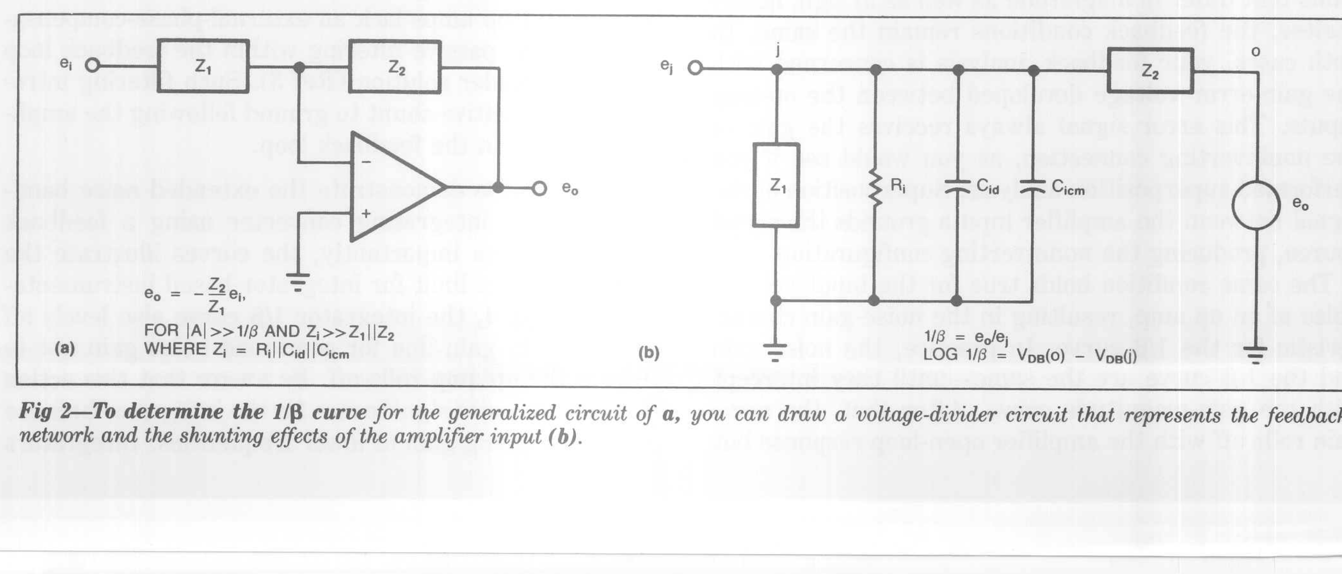

Fig 2—To determine the 1/β curve *for the generalized circuit of **a**, you can draw a voltage-divider circuit that represents the feedback network and the shunting effects of the amplifier input (**b**).*

indication of gain accuracy vs frequency and the ultimate bandwidth limit. Furthermore, the $1/\beta$ curve demonstrates that the circuit's signal bandwidth can be different from its noise bandwidth. Note that the previous feedback-network analysis returns Z_1 to ground as it would in a noninverting op-amp configuration, even though the op amp shown is in the inverting mode.

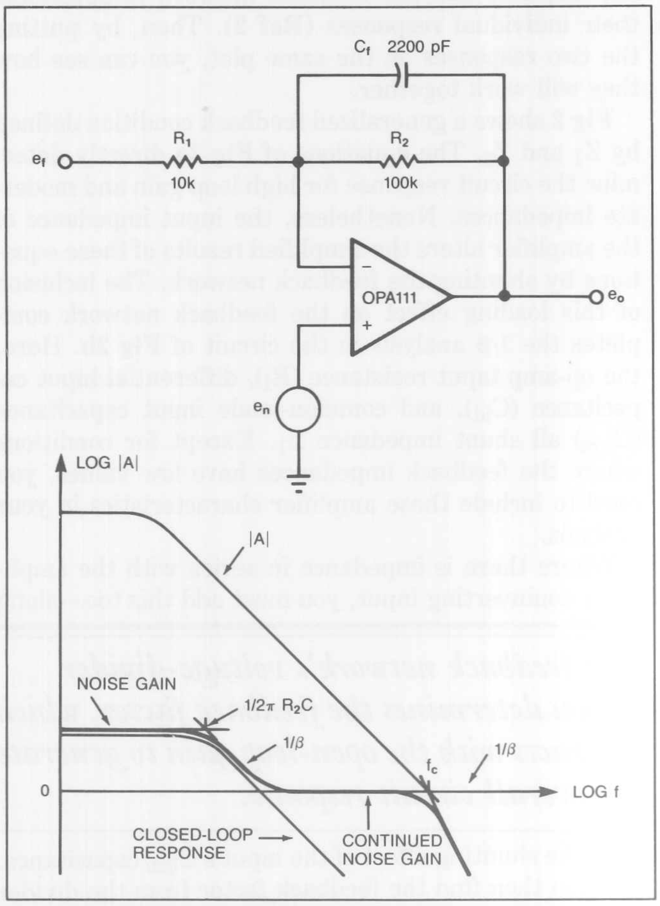

Fig 3—Highlighting the difference between closed-loop gain and noise gain, *this inverting op-amp configuration demonstrates the greater bandwidth that is often available to amplifier noise.*

Underlying the difference between noise and signal bandwidth is the concept of noise gain, which is the source of some of the more common op-amp application problems. For any given feedback network, the inverting and noninverting configurations develop signal gains that differ in magnitude as well as in sign; nevertheless, the feedback conditions remain the same. In both cases, your feedback analysis is concerned with the gain-error voltage developed between the op-amp inputs. This error signal always receives the gain of the noninverting connection, as you would see if you performed superposition analysis. Superposition of the signal between the amplifier inputs grounds the signal source, producing the noninverting configuration.

The same condition holds true for the input voltage noise of an op amp, resulting in the noise-gain characteristic for the $1/\beta$ curve. In practice, the noise gain and the $1/\beta$ curve are the same—until they intercept with the gain-magnitude curve. After that, the noise gain rolls off with the amplifier open-loop response but the $1/\beta$ curve continues on its path. For the noninverting voltage amplifier, the noise gain and the closed-loop gain, A_{CL}, are the same.

Watch out for surprises

In inverting configurations, this correspondence does not hold true, giving rise to frequent surprises during attempts at noise filtering. The simplest case of the inverting amplifier, where it is common practice to bypass the feedback resistor, serves to illustrate the inverting relationship **(Fig 3)**. Bypassing the feedback resistor is intended to limit noise bandwidth, and it does indeed remove noise presented as an input signal. However, the circuit will continue to pass amplifier noise across the entire op-amp bandwidth. C_f shunts the signal supplied through R_1 for the desired lowpass roll-off of the op amp's e_o/e_i response. To the op-amp noise voltage, e_n, C_f merely presents the unity feedback of a voltage-follower. Noise gain drops to unity but continues out to the open-loop roll-off of the op amp. This leveling off of $1/\beta$ also shows why the op amp must be unity-gain stable, even though the circuit gain has been rolled off well below the amplifier response. With $1/\beta$ following the unity gain axis, the critical intercept occurs at f_c.

While the continued noise gain is at a lower level, it covers much of the amplifier bandwidth, which can result in a dramatic increase in output noise. For example, if you're using the 2-MHz Burr-Brown OPA111 shown and choose C_f to obtain a 2-kHz roll-off, only 0.1% of the amplifier bandwidth will be enclosed in the intended system response. Although the logarithmic scale of the frequency axis may be visually deceptive, the remaining 99.9% of the bandwidth is still available to the amplifier's voltage noise. For an initial gain of 10, the output noise that this amplifier produces is more than doubled by the bandwidth effect. Many active-filter configurations are subject to the same limitation.

The only way to avoid excessive noise bandwidth is to restrict the frequency range of the op amp. By doing so, the control of the noise response switches from the $1/\beta$ curve to the amplifier roll-off. Where the op amp has provision for external phase compensation, this control is a simple matter and permits you to remove bandwidth from signal and noise alike. However, because most op amps lack an external-phase-compensation facility, passive filtering within the feedback loop offers a broader solution **(Ref 3)**. Such filtering introduces a capacitive shunt to ground following the amplifier but within the feedback loop.

You can also demonstrate the extended noise bandwidth of an integrating converter using a feedback plot but, more importantly, the curves illustrate the dynamic-range limit for integrator-based instrumentation. In **Fig 4**, the integrator $1/\beta$ curve also levels off at the unity-gain line for continued noise gain out to where the op amp rolls off. Be aware that this action has far less noise significance for the integrator because of its increasing gain at lower frequencies. Integrators

designed for operation to 1 kHz or even higher are generally unaffected by the added noise bandwidth.

Nevertheless, the feedback plot for the integrator demonstrates a unique bandwidth limitation involving two critical intercepts. Not only does the 1/β curve intercept the gain-magnitude curve at the high-frequency extreme, but it does so at the low-frequency end as well. Each intercept indicates a lack of amplifier gain for support of the feedback and a departure from the ideal response. At the high end, the 1/β curve and noise-gain level off, leaving A_{CL} to continue as long as the loop gain lasts. Next, 1/β intercepts the gain-magnitude curve at f_c where the noise gain rolls off.

This intercept is a high-frequency 3-dB point for the integrator response, which then usually rolls up rather than down. Upward response in this region is due to signal feedthrough caused by the feedback elements

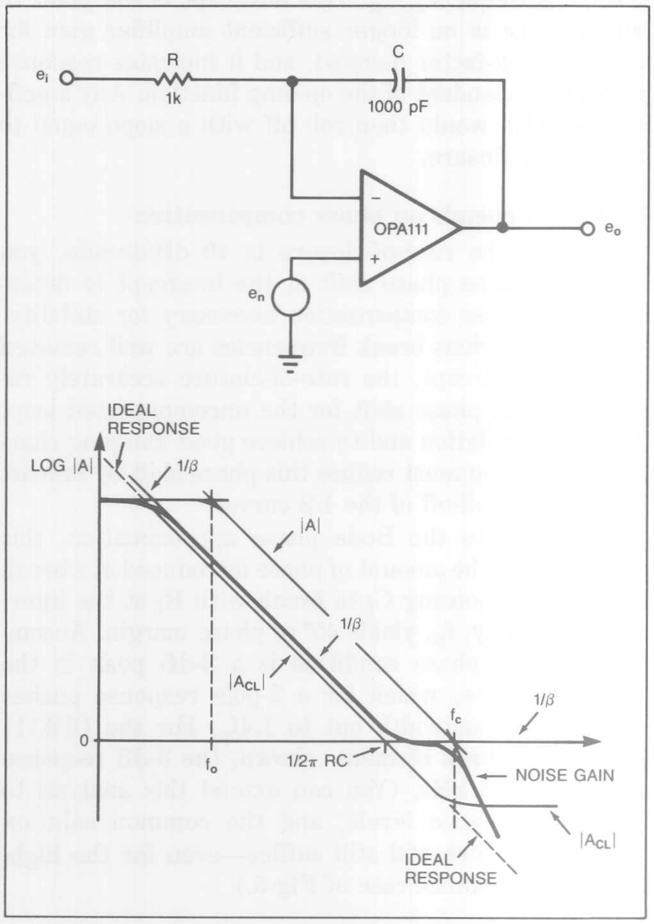

***Fig 4—Defining the dynamic range for integrating data converters,** the integrator 1/β curve displays upper and lower intercepts with the gain-magnitude response.*

in the absence of loop control. At the lower frequencies, the increasing gain demand encounters the dc gain limit of the op amp. This intercept marks the second 3-dB point for the integrator response, which sets the range for accurate performance. Both intercepts have a 20-dB/decade rate-of-closure, indicating stable operation.

Consider the gain error

Between the two integrator-response limits is the usable dynamic range for dual-slope A/D and V/F converters. The gain error limits this dynamic range; the plots are a graphic representation of this error. The gain error is inversely related either to a circuit's loop gain or the difference between the amplifier's open-loop gain and the feedback demand of 1/β. On the response plots, the gain error is the vertical distance between the two curves. For the **Fig 4** integrator, this separation decreases following 1/β's encounter with the unity-gain axis. From there, the separation finally reduces to zero at f_c. The gain error then becomes the distance between the dashed continuation of the ideal integrator response and the actual A_{CL} response. Graphically, this distance is the source of the large-signal limitation for integrating converters where higher signals correspond to the upper frequencies.

At the other end of the converter range, lower-level

For minimum-phase systems, you can directly read phase-shift information from the Bode plot of the op amp's gain-magnitude response.

signals demand low-frequency integrator operation that encounters a similar limitation. Below the frequency of the op amp's first pole, f_o, the separation between the 1/β and gain-magnitude curves again drops, signaling reduced loop gain. Moving further down in frequency, the 1/β curve finally crosses the op amp's dc-gain level, and the actual response flattens again. For integrating-type converters, this action defines a range of performance that is accurate to within 3 dB from f_c down to the lower intercept. To extend the dynamic range, you move the lower intercept downward either with a lower integrator-time-constant or with boosted dc gain.

A higher accuracy dynamic range results from the unique loop-gain conditions of the integrator. The loop gain is constant for the integrator from f_o to its unity-gain crossing. The gain error in this range is constant as marked by the uniform separation of the gain-magnitude and 1/β curves. You can compensate for such an error by making a fixed adjustment to the feedback network, leaving gain-accuracy bounded by the stability of the network. This limit permits you to adjust the more restricted dynamic range to 0.01% levels. For the OPA111 op amp and a 100-kHz integrator crossover frequency, this more precise dynamic range has a span of 100,000:1.

Feedback factor may introduce surprises

The previous discussion of the inverter and the integrator considered the feedback network independent of the amplifier input shunting. Although engineers frequently use this simplification, they often encounter unexpected results. Because of the feedback factor, most first-time users of op amps with large feedback-resistance values are surprised by the response curve. Transient-response ringing or even oscillation sometimes occurs; the common cure is a capacitive bypass

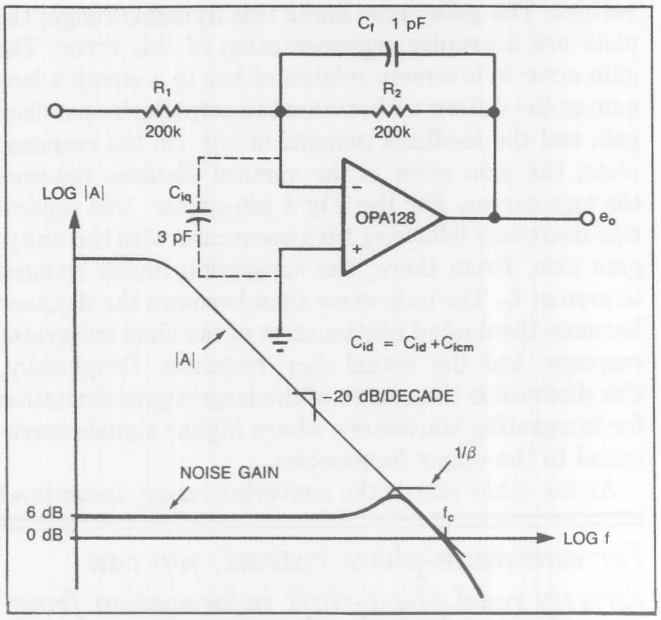

Fig 5—Higher feedback resistances will react with the op amp's input capacitance *to produce a peaking effect, which the 1/β curve anticipates.*

of the feedback resistor. The 1/β curve can display the problem and provide some guidance in the selection of the bypass capacitor.

Underlying the problem is the op-amp input capacitance's effect on the feedback factor. By including this capacitance with the voltage divider formed by the feedback resistors, you can achieve the results of the 1/β curve in **Fig 5.** This curve rises at high frequencies, increasing the rate-of-closure and flagging the need for closer stability analysis. The phase margin drops as 1/β rises and, at the limit, goes to zero if the 1/β rise spans two decades of frequency. Generally, the span is much smaller than that and the Bode phase approximation evaluates the actual conditions. The key to minimizing the effect on the feedback factor is the low input capacitance that the small input FETs of the OPA128 device provide. The net 3 pF of input capacitance leaves the response undisturbed until the parallel combination of the two resistors reaches 50 kΩ.

The capacitive bypassing of R_2 increases the high-frequency feedback, which counteracts the shunting of C_{ia} by leveling off the 1/β curve. The selection of this capacitor is better illustrated by **Fig 6**'s photodiode amplifier. You can reduce the nonobvious bandwidth of this application to an equation. The circuit contends with diode capacitances at the input that can reach 20,000 pF. As a result, the break in the 1/β curve is generally far removed from the intercept, making the rate-of-closure analysis accurate without requiring any adjustment of the phase-shift approximation.

Unfortunately, the bandwidth for the circuit of **Fig 6** is obscured by its function. Because its function is a current-to-voltage conversion, rather than simple voltage gain, you cannot draw the signal-gain curve on the gain-magnitude response to estimate bandwidth. When you inspect the circuit to ascertain its bandwidth limitation, you'll see that the only inherent break frequency is that of the feedback resistance and the capacitance of the input circuit. By plotting the 1/β curve, however, you can see that the loop gain remains to support the ideal feedback condition far beyond the f_i break frequency.

Initially, the 1/β curve is flat at unity because of the direct output-to-input connection of R_f. When the feedback is later shunted by C_D and C_{ia}, 1/β rises at a 20-dB/decade rate. The transition between these regions occurs at

$$f_i = 1/2\pi R_f(C_D + C_{ia}),$$

where $C_{ia} = C_{id} + C_{icm}$.

The intercept with the gain-magnitude curve marks the end of the response rise for the noise gain. This curve has a −20-dB/decade slope so, if left uncompensated, the rate-of-closure at the intercept will be 40 dB/decade. Thus, the plot indicates two poles at that intercept frequency, f_p. This intercept is the point at which there is no longer sufficient amplifier gain for the feedback-factor demand, and it indicates response roll-off independent of the op-amp function. Any amplifier function would then roll off with a slope equal to the rate-of-closure.

Stability depends on phase compensation

Because the rate-of-closure is 40 dB/decade, you should examine phase shift at the intercept to determine the phase compensation necessary for stability. When the various break frequencies are well removed from the intercept, the rate-of-closure accurately reflects 180° of phase shift for the uncompensated loop. To avoid oscillation and to achieve good damping characteristics, you must reduce this phase shift by at least 45° through roll-off of the 1/β curve.

According to the Bode phase approximation, this phase shift is the amount of phase introduced at a break frequency. Choosing C_f to break with R_f at the intercept frequency, f_p, yields 45° of phase margin. Accompanying this phase condition is a 3-dB peak in the signal response, which for a 2-pole response pushes the −3-dB bandwidth out to $1.4f_p$. For the OPA111 and the feedback elements shown, the 3-dB response extends to 48 kHz. (You can extend this analysis to lower capacitance levels, and the common solution mentioned above will still suffice—even for the high-feedback-resistance case of **Fig 5.**)

As long as C_f breaks with R_f at the frequency of the intercept, the 1/β rise contributes no more than 45° of phase shift. In the range where the op-amp phase shift is 90°, this rise leaves a stable 45° phase margin. Nevertheless, as the op amp approaches its crossover frequency, f_c, its contribution to phase shift moves toward 135°. The rule of thumb for selecting C_f remains valid, however, because any intercept near f_c must be a result of a 1/β rise of short duration. The added phase shift of the amplifier, accompanied by a necessary decrease in feedback phase shift at the intercept, results in a net zero effect. By simple sketching of the phase approximations for the 1/β and gain-magnitude curves, you can show this transition.

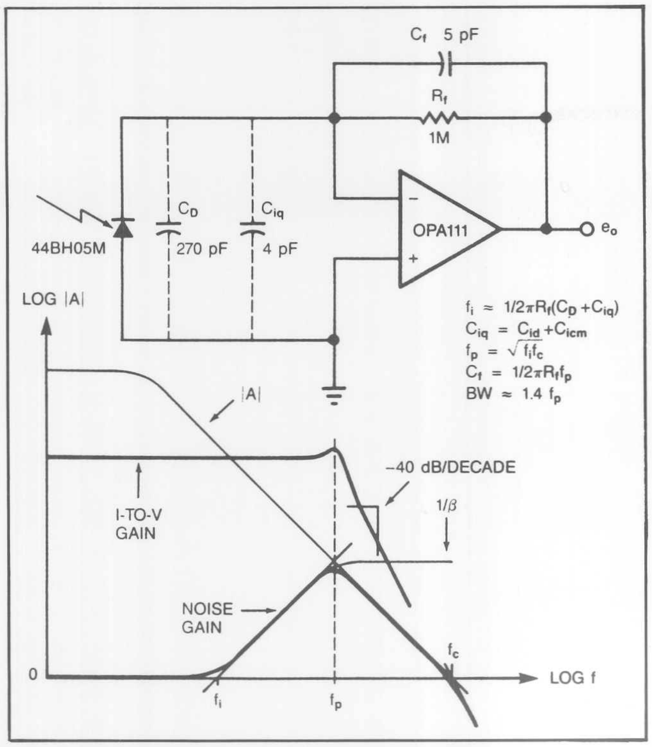

***Fig 6—A photodiode amplifier's current-to-voltage function** serves to obscure its bandwidth and stability, but you can rely on feedback-loop conditions to define its performance.*

Simple, yet elegant

To select the compensation capacitance, it is desirable to reduce the graphical analysis to an equation. Luckily, the response plots provide an elegantly simple solution. Straight-line extensions of the 1/β and gain-magnitude curves form a triangle with the horizontal axis. These extensions have equal but opposite slopes, which form an isosceles triangle. The peak of the triangle, located over the center of its base, lies at the average of the base end points. Mathematically, this average point is equal to

$$\text{Log } f_p = (\text{Log } f_i + \text{Log } f_c)/2.$$

Given the expressed logarithmic nature of the frequency axis, you can reduce this relationship to the simple geometric mean of the two characteristic frequencies:

$$f_p = \sqrt{f_i \times f_c},$$

where $f_i = 1/2\pi R_f(C_D + C_{id} + C_{icm})$, and f_c equals the unity-gain bandwidth of the op amp.

For a third input-circuit effect, the 1/β curve demonstrates stable conditions where typical gain and phase plots would point to oscillation. In addition to input capacitance, op amps have input inductance; this combination produces a high-frequency resonance. The inductance is small but inescapable, being associated with internal input wires and being compounded by external wiring.

When it comes to high frequency . . .

For very high-frequency amplifiers, like the OPA620 video amplifier of **Fig 7,** sufficient amplifier gain exists at the resonant frequency to give the appearance of zero gain margin. A comparison of the output signal (e_o) with that at the summing junction (e_j) produces the plot's gain and phase responses. Following unity crossover, the gain curve rises again above the unity axis; this rise generally guarantees oscillation for lower gain levels. Adding to stability concerns is the phase plot, which swings wildly through 180° during the gain peak.

By adding the 1/β curve to the plot, you can see that this curve does not intersect the gain peak but merely rides over it. Without an intercept there is no oscillation, regardless of the phase shift, because the loop gain is insufficient. Loop-gain demand rises in synchronization with the gain peak because the resonant circuit also alters the feedback network.

In many cases, the gain peaking results from conditions in the amplifier output rather than from the input circuit. In such a case, no corresponding modification of feedback occurs, and an intercept and oscillation result. However, for **Fig 7,** the gain margin remains high, as you can see by the separation between the 1/β curve and the gain response when the phase reaches 180°. This separation remains large throughout the region of higher phase shift, indicating good relative stability.

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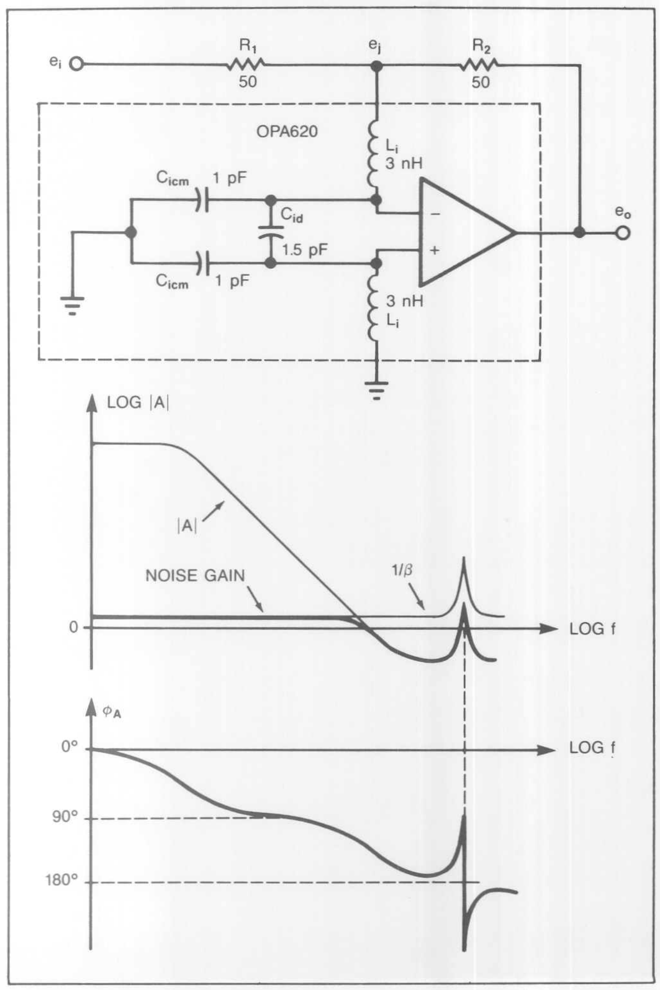

***Fig 7—Although amplifier gain and phase plots suggest instability,** the 1/β curve shows stable conditions for a circuit with input-lead inductance.*